Activity I: Proving the Pythagorean Theorem (Grade Levels: 6-9)

Standards:
Standard 7: Reasoning and Proof

Objectives:

The Pythagorean theorem can be proven using several different basic figures. This activity introduces student to two such figures with a brief explanation of how to go about the proof. The activity will demonstrate alternate solutions to the problems as well as provide a glimpse into the way early mathematicians reasoned about mathematics.

At the end of this activity the students should be able to:

- understand that there are many ways to approach a problem
- not to be completely reliant on a given drawing
- recognize that not all geometric proofs are two column deductive proofs so familiar in traditional textbooks.

Activity 1

This activity is a high school level activity that can be adapted for middle school and upper elementary students by simply having the students determine as many as possible Pythagorean triples.

Proving the Pythagorean Theorem

Pythagoras of Samos, c.560–480 BC, was a Greek philosopher and religious leader who was responsible for important developments in the history of mathematics, astronomy, and the theory of music. He migrated to Croton where he founded a philosophical and religious school that attracted many followers. Because no reliable contemporary records survive, and because the school practiced both secrecy and
communalism, the contributions of Pythagoras himself and those of his followers cannot be distinguished. The most important discovery of this school was the fact that the diagonal of a square is not a rational multiple of its side (that is, the diagonal of a square is not a number that can be expressed as the ratio of two whole numbers.). In essence, this showed the existence of irrational numbers. This discovery disturbed Greek mathematicians and the Pythagoreans themselves, who believed that whole numbers and their ratios could account for geometrical properties. Pythagoreans believed that all relations could be reduced to number relations (“all things are numbers”).

The Pythagoreans knew, as did the Egyptians before them, that any triangle whose sides were in the ratio 3:4:5 was a right-angled triangle. The so-called Pythagorean theorem, that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides, may have been known in Babylonia, where Pythagoras traveled in his youth. The Pythagoreans, however, are usually credited with the first proof of this theorem.

Much of the Pythagorean doctrine that has survived consists of numerology and number mysticism, and the influence of the belief that the world can be understood through mathematics. That belief was extremely important to the development of science and mathematics.

**Proving the Pythagorean Theorem**

The following figure is the typical figure used to prove the Pythagorean theorem that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. \( c^2 = a^2 + b^2 \)

1. If the right triangle ACB were isosceles, the figure would appear as follows.
Show how with the addition of three lines the proof of the theorem is as simple as $2 + 2 = 4$.

2. The following figure was used by President Garfield in proving the Pythagorean theorem. His method is based on the fact that the area of the trapezoid ACED is equal to the sum of the areas of the three right triangles ACB, ABD, and BED.

Prove the Pythagorean theorem using President Garfield’s method.

3. A Hindu mathematician named Bhaskara used the following figure to prove the Pythagorean theorem by showing the sum of the area of the small square and the area of the four congruent right triangles equal the area of the large square.
Student Activity Page
Activity 1: Proving the Pythagorean Theorem

Pythagoras of Samos, c.560–480 BC, was a Greek philosopher and religious leader who was responsible for important developments in the history of mathematics, astronomy, and the theory of music. He migrated to Croton where he founded a philosophical and religious school that attracted many followers. Because no reliable contemporary records survive, and because the school practiced both secrecy and communalism, the contributions of Pythagoras himself and those of his followers cannot be distinguished. The most important discovery of this school was the fact that the diagonal of a square is not a rational multiple of its side (that is, the diagonal of a square is not a number that can be expressed as the ratio of two whole numbers.). In essence, this showed the existence of irrational numbers. This discovery disturbed Greek mathematicians and the Pythagoreans themselves, who believed that whole numbers and their ratios could account for geometrical properties. Pythagoreans believed that all relations could be reduced to number relations (“all things are numbers”).

The Pythagoreans knew, as did the Egyptians before them, that any triangle whose sides were in the ratio 3:4:5 was a right-angled triangle. The so-called Pythagorean theorem, that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides, may have been known in Babylonia, where Pythagoras traveled in his youth. The Pythagoreans, however, are usually credited with the first proof of this theorem.

Much of the Pythagorean doctrine that has survived consists of numerology and number mysticism, and the influence of the belief that the world can be understood through mathematics. That belief was extremely important to the development of science and mathematics.

The following figure is the typical figure used to prove the Pythagorean theorem that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. $c^2 = a^2 + b^2$
1. If the right triangle ACB were isosceles, the figure would appear as follows.

Show how with the addition of three lines the proof of the theorem is as simple as \(2 + 2 = 4\).

2. The following figure was used by President Garfield in proving the Pythagorean theorem. His method is based on the fact that the area of the trapezoid ACED is equal to the sum of the areas of the three right triangles ACB, ABD, and BED.

Prove the Pythagorean theorem using President Garfield’s method.
3. A Hindu mathematician named Bhaskara used the following figure to prove the Pythagorean theorem by showing the sum of the area of the small square and the area of the four congruent right triangles equal the area of the large square.

Prove the Pythagorean theorem using Bhaskara’s method.
Activity 1: Proving the Pythagorean Theorem

Solutions

1. 

```
  b
 /   \
/     \\
/       \\
/         \\
/           \\
/             \\
/               \\
/                 \\
/                   \\
/                     \\
/                       \\
/-----------------------------------
   a
 /   \
/     \\
/       \\
/         \\
/           \\
/             \\
/               \\
/                 \\
/                   \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\
/                     \\n``` 

Proof: Consider the shaded regions as being moved from the small squares into the large square.

Moving all four such regions shows that: \( a^2 + b^2 = c^2 \)

2. 

\[
\frac{1}{2}c^2 + \frac{1}{2}ab + \frac{1}{2}ab = \frac{1}{2}(a + b) + (a + b)
\]
$$1/2(c^2 + ab + ab) = 1/2(a^2 + 2ab + b^2)$$
$$c^2 + ab + ab = a^2 + 2ab + b$$
$$c^2 + 2ab = a^2 + 2ab + b^2$$
$$c^2 = a^2 + b^2$$

3. The length of the side of the small square is \((b - a)\).
$$\frac{(b - a)^2}{2} + 4 \left(\frac{1}{2}ab\right) = c^2$$
$$b^2 - 2ab + a^2 + 2ab = c^2$$
$$b^2 + a^2 = c^2$$